Desert Island Physics

(for Physics 8)

(Calculus-based)

by

Philip Petersen, PhD

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**A. Preface:**

Imagine being stuck on an island with no companions, no books, no computer, and no cell phone. Then imagine that everyone in the world and all those assists were annihilated. Could you reconstruct the scientific description of our physical world we call Physics from memory? To do so, you would have to have a simple grasp of what physics is, with mental connections which work the way the mind does, not only in a logical, left-brained fashion, but also in a connective, right-brained structural way.

Calculus-based Physics is extremely complex, and the way it is presented in most texts is logical and left brained, with plenty of subscripts and complications that make it hard for student to learn the processes. As a theoretical physicist and cosmologist, I am always searching for a simpler way of representing the physics of the universe. This has led me to consider how to remember formulas and ideas, when I don’t have a book or computer with me. Some physicists call this ‘back of the envelope’ calculations, like those done in a restaurant. I call it Desert Island Physics.

As a professor, I have also realized that students don’t remember the ideas and formulas of physics very well. On a midterm exam without a cheat sheet, many of them do very poorly, and even if they do well, when the final comes along, they have forgotten many of the formulas, and misunderstood some of the ideas.

Long ago, as an undergraduate physics student, I found that putting key formulas and ideas on flash cards and memorizing them was very useful. Before using them, my physics exam scores were ordinary. After using them, I rose to the top of my class at the university. What was it that made a difference? Two things: 1. speed—I could write down formulas immediately; and 2. I could work from the formulas memorized to other formulas which were needed for problems.

This book will recommend memorizing Keystone Formulas, I call them, and Edible Ideas (one needs to eat and nourish on a desert island). If one uses flash cards or some other means of memorizing these things, most of basic Physics becomes transparent and easy. In addition to Keystone Formulas and Edible Ideas, **I will emphasize techniques for simplifying the amount written down in working problems, one of which is eliminating as many subscripts as possible, another is not to write down any term in an equation you already know to be zero.**

It amazes me is that current physics can explain everything that happens in the physical world on a daily basis. I jokingly say that Physics is the fundamental science—all other sciences are based on it. For example, Chemistry is the result of the understanding of Atomic Physics. Biology is the result of Chemistry. Geology is the result of Physics and Chemistry. Other Physicists may not agree, but even Anthropology and Psychology may be related to Chemistry (genetics) and the Physics of the brain and consciousness.

There are parts to that connection that are unproven, but someday Physics may provide us with a true Theory of Everything, not just the physical. Quantum Theory and its interpretations are leading the way to the understanding of how the world is related to human consciousness and the brain. The current Theory of Everything called the M-Theory (M is for multidimensional membranes) seems to work as an unproven theory of the physical world, and may have implications for consciousness as well, when combined with Quantum Theory.

**B. Introduction**

What is Physics?

I like to say that Physics **is the study of matter and energy, its structure and changes.**

It’s not enough to just say ‘matter’ for several reasons. Physics has become enamored with the **field** concept (*a presence in space that creates a force*), and one could describe that presence as energy. For example, the electric force is ascribed to virtual **photons** (*light particles*) in a presence around a charged particle. Another reason we should include energy in the study of physics is that now we know the universe is accelerating in its expansion because of a **dark energy** in space.

**Condensed Matter Physics** (*liquids, solids, liquid crystals, etc*) is involved not only in understanding the structure of matter, but in understanding how and why it changes, for example, a phase transition: as when ice melts. So including changes as part of the study of Physics involves more.

Physics (and all science) is based on the Scientific Method. To stir up an atmosphere of controversy and critical thinking, I sometimes describe science as the most powerful religion in the world. After all, more people believe in science than anything else, and it brings people together with a common language, an ethically important thing. The foundational beliefs of the ‘religion’ of science, are the postulates of the Scientific Method.

**The Scientific Method**, begun by the Greeks (Aristotle) and completed by \*Francis Bacon in 1620 in his book, Advancement of Learning, is a set of rules of the game for **science.**

It involves the following steps:

1. Focus on a *problem to be solved*

2. Gather *information*.

3. Formulate a **hypothesis.**

4. *Objectively test* the hypothesis by making predictions and performing experiments or making observations.

5. Draw a *conclusion.*  (Is the hypothesis true or false? If false, reformulate it. If true under all possible circumstances, the hypothesis is elevated to the status of a **theory**. This does not mean it will never change. Law is not a good term: we expect science to change.)

**Paradigm shift**: Thomas Kuhn (1962) describes it as an ‘intellectually violent revolution… where one conceptual world view is replaced by another’. He claimed that science does not evolve, but rather undergoes paradigm shifts.

**Falsifiability**: Karl Popper (1963) claimed that a good theory should be falsifiable, or refutable, testable. In this way he hoped to eliminate ‘pseudoscience’. This is still under debate. Suppose I make the hypothesis that ‘God Exists’. We don’t know what to test scientifically to disprove it, so it is a ‘bad’ hypothesis. However, that doesn’t mean that someday we might not have scientific tests for the existence of God.

Physics 8 portion of book:

**Thermodynamics.**

**17. Temperature and Heat.**

**Heat** is an amount of energy in a substance in the form of molecular motion, or kinetic energy. It is ‘**global**’, or contained in a whole mass. Heat is measured in Joules or calories.

 **1 calorie = 4.187 Joules** (Physics calorie)

 **1 Calorie = 1 kilocalorie = 1,000 calories**. (Food Calorie)

**Think**: If you consume 2,000 Calories in a day, how many Joules of energy is that equivalent to?

 A joule is about the energy of 10 million flea push-ups.

**Temperature** is a measure of the average kinetic energy in a location.

 It is ‘**local**’. (Think of a weather forecasting map. Each city has a different temperature.)

**The three temperature scales**:

**Farhrenheit, Celsius, and Kelvin**. T (Kelvin) = T(Celsius) + 273.16.

**Think**:

 There are 180 F degrees and 100 C degrees between the freezing and

 boiling point of water. Freezing is 32o F and 0o C. 180/100 = 9/5.

 Therefore,T(F) = (9/5)T(C) + 32, and the reverse conversion can be

 found from it by algebra.

**Absolute zero** is zero Kelvins (no motion of the molecules except

 for a quantum amount). This is equivalent to -273.16o C. Thus

 Heat depends on mass, temperature and specific heat.

 **Specific heat**, c, for a substance is the amount of heat necessary to

 raise one gram of it 1 degree C.

**Expansion**: Heat expands, cold contracts. Some substances expand more when heated. Differing **coefficient of linear expansion**. **Thermostat**.

Thermal expansion of wire: ∆L = αL0∆T .

Thermal expansion of volume: ∆V = βV0∆T .

**Tensile Stress:** (for bound rod) F/A = -Yα∆T .

**HEAT TRANSFER (3 Ways)**

**1. Conduction**: transfer of Kinetic energy by collision of molecules, atoms,

 or other particles. Heat current, H = dQ/dt = kA∆T/L.

 Heat Q = mc∆T = nC∆T, c = specific heat, C = specific heat/mole.

**Metals** are the best conductors of heat and electricity , since they have ‘loose’ electrons.

**Insulators**: poor conductors--outer electrons in the atoms are firmly attached. Liquids and gases are usually poor conductors.

Heat is transmitted from high temperature to low.

**Newton’s Law of Cooling**: **The rate of cooling ~ ΔT**, the temperature difference at the boundary.

**Think**: For keeping a cup of coffee warm, is it better to add the cream right away or right before drinking?

**2. Convection**: heat is conveyed by currents in liquids or gases. Often this happens because a heated parcel of fluid rises and cools then sinks.

**Think**: In a house fire it is best to keep low as you leave the building for two reasons. What are they?

**3. Radiation**: transfer of energy in the form of light. Heat radiation or infrared is a form of light. Light can be radiated even in empty space.

 The sun is a good radiator.

 Stephan-Bolztmann relation: H = AeσT4 , σ = 5.67 x 10-8 W/m2K4 .

 The process of radiation involves emission and absorption.

**Emission** is when light is given off by a substance.

**Absorption** is the taking on of energy by an electron, atom, or molecule.

**A black body** is a perfect emitter and absorber.

 **Think**: Which would you rather wear on a hot day: white or black?

 **The Greenhouse Effect**: ultraviolet radiation hits the earth and is converted into heat. Carbon dioxide, water molecules, and methane act like a blanket and hold the heat in the atmosphere (Global Warming?).

**Solar Power**: Energy radiated from the sun--about 1387 watts of power hit every square meter of the earth (adjusted by the angle the surface and the absorption of the cloud cover).

**Think**: If a bank of solar cells is 40% efficient, how many 100 watt light bulbs could a square meter of cells light?

**18. Thermal Properties of Matter.**

**CHANGE OF STATE** Q = mL

**Melting** **point**: the temperature at which a solid is converted to a liquid.

**Latent Heat of fusion**: the heat necessary to melt a solid. This is L = 80 cal/g for ice.

**Boiling point** is the temperature at which a liquid boils.

**Latent Heat of vaporization**: the heat necessary to vaporize a liquid. This is L= 540 calories/g for water.

**Think:** When you get out of a swimming pool, you very cold. Why?

**Think**: Water droplets in the air ‘condense’ when the air gets cool enough.

 Water molecules stick to dust and forming drops when their kinetic energy is too low to bounce off. When **Brownian motion** is overcome by weight, rain falls.

**Brownian motion** is the random buffeting of suspended particles by air molecules. This is seen as dust suspended in a sunny window.

Boiling point also depends on pressure. Water boils at a lower temperature in the mountains, where the air pressure is lower. The vapor has a higher relative pressure and escapes. The lower temperature means that it takes longer to boil an egg, for example.

Other phase transitions occurring at a **transition temperature**: magnetic at the **Curie Temperature**, crystal, liquid crystal, and quasi-crystal.

**Ideal Gas Law:** for no intermolecular interaction, pV = nRT, n =mtot /M, M is mass/mole, R = 8.31 J/mol K. This is an example of an **Equation of State and State Variables** (in this case p, V, and T).

Van Der Vaals Equation: expresses interaction of molecules when they get close.

Root mean square velocity (measurable), vrms = √(3kT/m).

Mean Free Path, λ = V/(4π√2r2N)

Molar Heat Capacity of ideal monatomic gas CV = (3/2)R, diatomic: CV = (5/2)R ,

Solid: CV = 3R.

Maxwell-Boltzmann velocity distribution of an ideal gas:

f(v) = 4π(m/2πkT)3/2 v2e-mv2/2kT .

**19. The First Law of Thermodynamics.**

**The First Law:**  **Internal Energy** ∆U = Q – W, or differentially, dU = dQ-dW,

Where Q = mc ∆T = heat added to system (+ive), and W is work done by the system **on the environment**(+ive). W =∫pdV .

Simple processes:

**Isobaric = constant pressure 🡪** W = p ∆V

**Adiabatic = no heat exchange, Q 🡪** ∆U= - W Ex. Free (**adiabatic**) expansion.

γ = Cp/Cv , TVγ-1 = constant, pVγ = constant.

**Isochoric = constant volume 🡪** W = 0

**Isothermal = constant temperature 🡪** if ideal gas, U is proportional to T (not p or V), ∆U = 0 and Q = W.

For an ideal gas, Cp (constant pressure) = Cv (constant volume)+ R.

**20. The Second Law of Thermodynamics.**

**The Second Law (4 versions):**

**1. Heat flows spontaneously from hot to cold, not the other way around.** Clausius.

**2. Heat cannot be converted completely into work.** The efficiency of a heat engine or refrigerator is never = 100%. Efficiency, e = W/QH = 1 – |QC/QH| And 2nd Law is that e never reaches 1.

Otto Cycle (automobile) = 1 – 1/rγ-1, where r is the compression ratio (Volume ratio—max to min).

Carnot Cycle (most efficient) e = 1 - TC/TH. Must be Kelvin temperature.

**3. Entropy** (∆S = Q/T at a constant temperature) **always increases for a macroscopic closed system).** ∆S≥0. ∆S = ∫dQ/T.This is the same for all processes leading between two states. Thus we can choose a process which is easy to calculate (like isothermal, T =const.).

**4. Information:** S =k ln w**,** where w is the number of microscopic states.

**32. Electromagnetic Waves.**

Maxwell’s Equations–Electricity and Magnetism –> electromagnetic waves move at the speed of light. B = ε0μ0cE and Max. Eqns. 🡪 c = 1/√( ε0μ0) , a fact which Hertz tested experimentally a few decades after Maxwell derived it. Therefore, **light is an electromagnetic wave**, from the classic viewpoint**.**

**The Wave Equation:** ∂2E(x,t)∂x2 = ε0μ0∂2E(x,t)∂t2

**Wave functions:** E(x,t) = E0sin(ωt-kx) and B(x,t) = B0sin(ωt-kx).

**Power:** P = loop∫**S∙**d**A,** where **S = (**1/μ0) **E X B**, the **Poynting Vector.**

Avg **S** is called the **Intensity**, I = E0B0/2μ0 .

**Radiation pressure:** p = I/c, for absorbed, 2I/c for reflected.

Electromagnetic waves (if contained) can be standing waves for E and B like waves on a rope. **E** and **B** are represented in the Hertz format as perpendicular, and this light is a transverse wave. However, Nicola Tesla claimed that it could be represented as a longitudinal wave, and this is also true mathematically (though almost never considered because light has no medium like sound—unless somehow the dark energy in space enters in).

Not all light is visible. **The electromagnetic spectrum:**

 **Radio Microwave Infrared Visible Ultraviolet X-Rays Gamma Rays**

**-----------------------------------------------------------> Energy, frequency** (E = hf)

**Wavelength** <**-----------------------------------------------------------------------------**

**A nanometer** is a billionth of a meter.

Visible light has a range of **400-700 nanometers**.

The **speed of light** in a vacuum or air is c = 186,000 mi/s = 2.99 x 108 m/s.

Speed of light was first accurately measured by Fizeau with a rotating, toothed wheel.

**Transparent** materials allow light to be passed on by vibrating electrons. A chain of absorptions and re-emissions occurs. This slows the passage of light and the speed of light is thus smaller in a dense medium. **Opaque** materials absorb all the light, turning it into random kinetic energy. Metals are opaque because their free electrons dissipate order by colliding.

The **eye** involves an adjustable lens which focuses an inverted image on the retina. The brain makes the image erect. Rods record black and white, and cones color.

The color of an object results from **selective reflection**.

The sky is blue because of **preferential scattering of blue light**.

The **additive primary colors** (light) are red, yellow, and blue.

**Subtractive primary colors** (pigments) are yellow, magenta, and cyan.

c = f λ --The **light (and wave) velocity formula**.

**Optics.**

**33. The Nature and Propagation of Light.**

**Reflection and Refraction**

**Geometric optics** assumes light can be represented by **rays**, ignoring interference. This is for wavelength λ << size of optical components.

**Reflection** occurs from surfaces whose electrons are free to oscillate along the surface and re-emit the light. That’s why metals are shiny and mirrors are silvered.

**The law of Reflection: the angle of incidence = the angle of reflection.**

These angles are measured for the incident and reflected ray of light relative to a line perpendicular to the surface called the **normal**.

Flat mirrors create **virtual images**. These images are not formed by light, but appear to be extensions of reflected rays. **Real images** are formed by actual light rays. They are formed by **concave mirrors** as in a Newtonian telescope.

**Refraction** is the bending of light as it enters a new material at an angle to the normal. The change in speed of light is causing the effect.

The **index of refraction** is n = c/v, where v is the speed of light in the medium. **Snell’s Law** gives the relation between the incident and refracted angles between two media:

n1sinθ1 = n2sinθ2.

However, the wavelength of light in a material is λ = λ0/n.

**Total Internal reflection:** the **critical angle** is for sinΘcrit = n2/ n1. There is a **critical angle** for refraction beyond which no light goes into the second medium–it is all reflected. This happens in **fiber optics**.

**Dispersion:** n= n(λ) . For a given medium, n varies slightly with λ. This is what disperses the light from the sun going through a prism.

**Polarization**—the direction of the Electric field of photons.

For going through a linear polarizer, **Randomly polarized light** is diminished in intensity I 🡪 I/2, where I is proportional to E2.

Linearly polarized light is diminished by I0 🡪 I = I0cos2Θ, the **Law of Malus**.

**Polarization by reflection:** light reflected at a surface is completely polarized at the polarization (Brewster) angle, Θp = tan-1(n2/ n1). Critical angle involves sin, polarization involves tan, same formula otherwise.

**Circular and elliptical polarization** involves a rotating E field.

The law of reflection and Snell’s Law can be derived from **Huygen’s principle** which says that each point on a wave from emits a circular wave front. The resulting wave front is a combination of those.

**34. Geometric Optics and Optical Instruments.**

**Mirrors** form images by reflection.

**Plane mirror:** using law of reflection to make a **ray diagram**, we find that the **lateral magnification**, m = y’/y = +1. **+ sign is erect image, - sign is inverted**.

**Spherical mirror:** (for **paraxial ray approximation**—rays nearly parallel to axis or R large) 1/s + 1/s’ = 1/f, where s = object distance, s’ = image distance, **f =** **focal length** (rays from infinity will focus at that distance from the vertex of the mirror)

If R is the **radius of curvature**, f = R/2.

**Magnification,**  m = y’/y = -s’/s.

**Petersen’s Rules** for signs of optical distances for s, s’, f, and r: real images produce +ive, virtual images –ive. For m, erect is +ive, inverted is -ive.

**Refraction at a spherical surface:** n1/s + n2/s’ = (n2 - n1)/R. Note that if R🡪∞, we have a flat surface and n1/s + n2/s’ = 0.

You can use the above equation applied twice to derive:

**The thin lens equation** (the same as for paraxial spherical mirrors):

1/s + 1/s’ = 1/f. Petersen’s rules for signs also apply.

**The lense-makers equation:** for grinding two sides with radii R1 and R**2,**

1/f = (n-1)( 1/R1 - 1/ R**2).**

**Three principal rays for ray diagrams:** 1. Ray parallel to initial axis.

2. Ray parallel to final axis. 3. Ray through the center.

A **mirage** is due to refraction of light from a source passing through air which is hotter closer to the ground. The speed of light varies with the temperature of air, and thus the angle of bending changes gradually. This curves the light so the image appears to be on the ground.

**Rainbows** come from light refracted differently for different colors (**dispersion**). This refraction happens as light passes through water droplets.

**Optical Instruments**

**A simple camera** is essentially a single converging lens producing an inverted image on photo paper or a digital receptor. The lens of the eye works similarly.

**f-stop** = f /D, the focal length of the lens of a camera over the diameter. However, when f-stop = 2, for example, we say f/2 (mathematically strange). The iris changes D, changing the f-stop. The amount of light hitting the lens depends 1/f2.

**A magnifier:**  has angular magnification, M = Θ’/Θ = s(near point=25 cm)/f.

**Microscope:** M = s(near point)s1’/f1f2 .

Lenses have **chromatic aberration** (different n and thus refraction for different colors) and **spherical aberration** (a perfect lens would be ground with parabolic surfaces). Spherical mirrors have only spherical aberration. More expensive cameras and optical devices correct for these with added lenses or parabolic grinding.

**Telescopes** are **refractors** with 2 lenses or **reflectors** (Newtonian) with 1 mirror and 1 lens. Both have **magnification**, m = -F/f.

The **angular resolution, R** of an optical device (like a telescope) with an **aperture** (opening) of diameter, D is

R = 1.22 λ/D.

Smaller is better for resolving smaller details.

**35. Interference.**

**Monochromatic** (one λ) and **coherent** (in phase, same φ) light, like from a laser, can be used to gauge the effects of interference.

Amplitudes add in **constructive interference:**

 Path difference: δ = r2 – r1 = mλ, m = 0, +-1, +-2, +-3, etc.

Amplitudes cancel in **destructive** **interference:**

 **Path difference:**  δ = r2 – r1 = (m+1/2)λ, m = 0, +-1, +-2, +-3, etc.

**Young’s 2 slit experiment:**

dsinΘ = mλ, as above, for light maxima fringes.

 dsinΘ = (m+ 1/2)λ, as above, for light minima. (d = slit spacing)

 Also, y = Rmλ/d where R is distance to screen, constructive fringes.

 Intensity I = I0 cos2(πyd/λR).

**Thin film interference**—the path difference for bouncing off two successive surfaces: 2t = mλ for constructive interference, m = 0, 1,2,…

 2t = (m+1/2)λ for destructive interference.

This is for the case of **0 or 2** **phase shifts** (of (1/2)λ) related to going into a higher index of refraction, n.

For **one phase shift** at only one surface, constructive and destructive interference formulas are switched.

**36. Diffraction.**

The same formula holds for the **minima** (dark fringes) for **one slit, Fraunhofer distant screen) diffraction** as for the **maxima** (bright fringes) for **two slit and multi-slit (diffraction grating) interference**. The formula is:

 sin(Θ) = mλ /d,

where theta is the angle off the center line going from the slit midpoint to the screen, m = ±1, ±2, ±3, etc., lambda is the wavelength of monochromatic light used, and d is the slit width for the one slit and the slit spacing for the two slit (or multi-slit) experiment.

**Intensity:** I = I0 [sin(β/2)/β/2]2, where β = (2πdsinΘ)/λ.

**Width of the single slit maximum:** Θ = λ/d .

**For a circular aperture (like telescope):** Θ =1.22λ/d .

**Diffraction grating:** same formula as two slit, except maxima are narrower and more well defined, making this device excellent for providing spectra, or spreads of wavelengths.

**Bragg X ray diffraction** —light bounces off two successive layers of a crystal of spacing d, and constructive interference occurs for path difference

2 d sin(Θ) = mλ, m = 0, 1, 2,...

This is a way of determining the crystal spacing, d.

**A hologram** is a wave pattern formed by the interference of monochromatic, coherent light scattered from a 3-d object, and light coming directly from the source. Information about the whole is contained in every part of the pattern.

**Modern Physics.**

**37. Relativity.**

**SPECIAL RELATIVITY:**

**Postulates:**  Forall **inertial frames of reference** (non-accelerating, the pendulum hangs down),

1. The laws of physics can be experimentally found to be the same (never found to be false).

2. The speed of light is the same, and independent of motion of the source. (experimentally verified).

**Mathematical Consequences:**

1. **Time dilation:** ∆t = γ ∆to, where t is time, to is **proper time (**on board), and gamma is the **boost factor.**

γ = 1/√[1-u2/c2], always greater than 1 (u is the relative velocity of 2 frames of reference).

2. **Space (Lorentz) contraction:** L = Lo/γ. Note that space and time expand and contract inversely, space compensating for time, and time compensating for space. That is why Special Relativity considers **events** happening in space-time, a unified 4-dimensional picture.

3. **Mass dilation:** m = γ mo, the same as the way time dilates. m is the **boosted mass**, and mo **the rest mass**. Note that this changes momentum to **p** = γ mo **v**, and Newton’s 2nd Law to **F =** d(γ mo **v**)/dt.

4. **Universal speed limit = c** for an object with mass, because m (inertia) asymptotically approaches infinity as v 🡪 c.

5. **E = mc2**, where **m is the boosted mass**, & can be converted into energy and energy into mass. This is derived from W = ∫Fdx = ∫[ d(γ mo **v**)/dt]dx by calculus.

This equation initiated the nuclear age.

Another vital equation can be derived: E2 = (m0c2)2 + (pc)2 . It is useful in generating the relativistic quantum equation. A photon has no rest mass, therefore E = pc.

**Lorentz transformations:**

x’ =γ(x-ut), y’ = y, z’ = z,

t’ = γ(t-ux/c2)

Addition of velocities does not work in a Galilean fashion. For high speeds we need (from Lorentz Transformations):

 v = (v’ + u)/(1 + uv’/c2), where u is relative speed of two observers.

Note that prime and unprimed may be interchanged by changing the sign of u.

**Tachyons:** for v>c, set (v/c)2 = 1 + α (real, positive), then time dilation and mass become imaginary and negative.

**Doppler shift:** f = f0√[(c-u)/(c+u)] for source moving away from observer. Change sign of u for source moving toward.

**Correspondence principle:**  when v << c, Newton’s Laws apply because m = m0 .

**GENERAL RELATIVITY**

**A. Local equivalence principle: Gravity and acceleration are locally equivalent**, as long as you are confined to observing within an infinitesimally small space.

**B. Space-time curvature and acceleration equivalence**: a product of the math of curved spaces.

A and B imply **gravity is equivalent to curved space-time**. This is the basis of **Einstein’s Field Equations**. They provide physics for all observers, accelerated or not—that’s why it’s called General Relativity.

Special Relativity is a limiting special case of General Relativity for v = constant.

**General Relativistic Metric (GRM):**  ds2 = gμνdxμdxν  in 4-D (repeated indices are summed). gμν = gνμ , a symmetric tensor, thus having 10 unique components. For a coordinat transformation, gμν ‘= (∂xα/∂xμ’)(∂xβ/∂xν’) gαβ .

gρσgστ = δρτ , where δρτ = 1 for ρ = τ, 0 otherwise.

Covariant vectors have lower indices and contravariant vectors have higher. Transformations between them: Aμ = gμνAν , and Aμ =gμνAν . Many tensors can be multiplied like matrices.

**Special Relativity Metric:** for v = constant, ds2 = c2dt2 - dx2 -dy2 - dz2 .

**GRM 🡪 Field Equations (10 by μν symmetry):**  Rμν = Λgμν – K[Tμν –(1/2) gμνT] , where Rμν is the curvature tensor of space-time, Λ is the cosmological constant, and Tμν is the matter energy tensor and T = gμνTμν , K = 8πG/c2.

**The Schwarzschild Metric (non-rotating, spherically symmetric black hole):**

Solution of Field equations:

ds2 = c2 (1- 2Gm/c2r)dt2 - dr2/(1-2Gm/ c2r) –r2(dΘ2 + sin2Θdφ2)

Note that the dr term blows up for

 r = rSch= 2GM/ c2 = 3km(M/Msun) .

Here time is overpowered by space and is relatively infinitely dilated relative to a distant observer. This is the radius of the **event horizon (Schwarzschild Radius)**, where escape speed becomes the speed of light. This also can be derived from E + U = 0, (1/2)mv2 - Gm’m/r = 0, setting v = c.

Proofs of General Relativity: Mercury’s orbital ellipse turns in sun’s gravity, light bends around the sun, time is dilated in a stronger gravitational field (called **gravitational time dilation**).

General Relativity is needed for dealing with strong gravity (like Black holes), but does not hold for the subatomic realm where Quantum Theory rules.

**Does tidal force of black hole rip you apart?** Classic tidal force F ≈ (2d/r3)Gmm’.

For a mass of 2 million solar masses = 2 x 106 x 1.9 x 1030 kg, the tidal force at the event horizon is 0.328 N. For smaller black holes, the force can become way larger, for larger bh’s smaller. So, **supermassive black holes** do not rip you apart at the event horizon, but at the center called the singularity, gravity can become very strong and crunch you unless the black hole is rotating rapidly, in which case a **wormhole (bh🡪white hole)** is possible.

**38. Quantum Physics I: Photons, Electrons, and Atoms.**

**Planck’s formula** (derived from the study of black-body spectra) is E= hf = hc/$λ$. = 6.63 x 10-34 J∙s, **Planck’s constant.**

Making the assumption that photons come in bundles or ‘quanta’ of energy as indicated in the formula, Planck was able to derive the **continuous spectrum formula** (blackbody curve): I(λ) = 2πc2/[λ5(ehc/λkT-1)].

**The Photoelectric Effect:** if photons come in bundles of energy equal to hf, then the maximum kinetic energy of electrons removed from a metal’s surface for monochromatic bombardment is Kmax = hf – φ , where φ is the work function (work to remove electrons from that surface). The stopping potential,V0, thus is related to wavelength of the photons:

 eV0 = hc/λ – φ.

**Atomic Line Spectra:** From experiment, Balmer found that atomic hydrogen lines were represented by transitions between energy levels,

 En = - 13.6 eV/n2 , where 1 eV = 1.6x10-19 Joules.

This was later derived by Neils Bohr from a theory of quantization. Other atoms are more complex, but still have levels related to n.

Photon energy may undergo **absorption** when photon energy is absorbed to raise electrons to higher energy levels, **emission** when an electron drops to a lower energy level. Absorption lines are dark lines against a continuous (blackbody) spectrum. Emission lines are bright lines.

**The nucleus:** Rutherford scattered alpha particles off gold foil and discovered that the scattering was minimal, indicating the positive charge was concentrated in a very small space we call the nucleus. (If a nucleus is the size of a marble and an electron the size of a grain of salt, the grain of salt would be 2 miles away.)

**The Bohr Model:** He assumed that angular momentum of the electron orbiting the Hydrogen nucleus was quantized. mvr = nh/2π. From that and classical physics, he derived the formula above for H spectra. He also got that the orbital radii r were proportional to n. All this can be derived in later quantum theory by assuming that the electron is a standing wave with wavelength proposed by DeBroglie: λ = h/mv.

**39. Quantum Physics II: The Wave Nature of Particles.**

Planck found that light behaved like a particle called a **photon** of energy

E = hf =hc/λ.

He needed this to describe a **continuous or black body spectrum**.

DeBroglie suggested that particles might behave like waves:

 λ = h/p,

where p = mv = momentum. h = 6.63 x 10-34 J s, **Planck’s constant**. Davisson and Germer proved this experimentally by interfering electrons in the same way as light, after accelerating an electron through a potential, V, λ = h/p = h/√(2mqV).

**Heisenberg Uncertainty Principle**: δx δp ≥ h/2π orδE δt ≥ h/2π, where deltas represent uncertainties.

**Electron Microscope:** uses wave properties of electrons instead of light to probe the very small. As speed of electron goes up, wavelength goes down and improves resolution.

**The wave function:** Ψ = Ψr + iΨi , a complex function.

Ψ\* = Ψr - iΨi , Probability of finding a particle in space dxdydz in time t is Ψ\*Ψ. The true nature of particles is waves, NOT wave-particle duality. A particle may behave like a particle in an experiment, but it is actually a **wave packet:** shaped like a pulse wave: Ψ(x) = ∫0∞A(λ)cos(2πx/λ)dλ, where A(λ) is sharply peaked around a the DeBroglie wavelength. This is one way of thinking of the Uncertainty principle: for a given momentum, there is a spread of positions where the particle might be found.

**40. Quantum Physics III: Quantum Mechanics.**

**The Particle in the Box (simplest QM problem):** using a particle in an infinite 1 dimensional well as a standing wave of wavelength, λ = h/p, one can propose Schrodinger’s Equation Time Independent Equation. Like a string attached at two ends, over a length, L. The standing waves are described by:

ψn = A sin(2πx/L), where the wavelength λ = 2L/n = h/p (DeBroglie), n = 1,2,3,… .

Thus, p =nh/2L, and since E = p2/2m, En = (1/8m)(nh/L)2.

However, the sine function is a solution to - d2ψ/dx2 = Cψ, where C is a constant.

We now can make the equation have units of energy if Ad2ψn/dx2 = En ψn.

We now let the left side be an operator, called the **Hamiltonian,** H. H ψn = En ψn.

H = pop2/2m +U (U = 0 , in this case) 🡪 pop = i(~~h~~/√(2m))∂/∂x , where ~~h~~ = h/2π.

If we add a potential to this for other problems, H = pop2/2m + U.

Then the **Time Independent Schrödinger Equation** is:

 (~~h~~2/2m) d2ψ/dx2 + Uψ = Eψ .

Individual solutions might be ψn and have energy **eigenvalues** En .

Solutions need to be normalized so the probability of location in all space is 1. ∫-∞∞ψ\*ψdx = 1. For a **Stationary State** (constant energy E) Ψ = ψ(x)e-i(2πE/h)t, so that E becomes and operator and ψ🡪Ψ in the **time dependent Schrödinger Equation**. E = i~~h~~∂/∂t and all derivatives become partials--

 (~~h~~2/2m) ∂2Ψ/∂x2 + UΨ = (i~~h~~∂/∂t)Ψ .

Remember also that there is an expanded version in 3 dimensions which is necessary for atomic physics, for example. That’s how we get 3 more quantum numbers because the wave function is separable in spherical coordinates. Ψ(**r**) = R(r)Θ(θ)Φ(φ), yielding 3 equations.

**Quantum Tunneling:** classically speaking, if you don’t have enough energy to get out of a valley and over the hill, you can’t. However, quantum theory says that in a nucleus for example, an alpha particle could break off without having enough energy--by tunneling through a potential barrier. This allows for things like Josephson Junctions, Diodes, LED’s, etc. There is a small probability (extremely small) you could run into a wall and tunnel through to the other side. We have never heard of that happening, or have we?

**The Quantum Harmonic Oscillator:**  two atoms in a diatomic molecule like O2 act like they have a linear spring between them with Force = -kx. The oscillatory potential energy between them is thus U(x) = (1/2)kx2. Plugging this into the one dimensional time independent Schrödinger Equation and applying boundary conditions, we get the energy eigenvalues

 En = (n + ½)~~h~~ω, ω =√(k/m) , n = 0, 1,2,3,… .

This can be done in three dimensions as well for solids, for example.

**The Quantum Rotator:** a diatomic molecule can rotate too. The angular solutions to rigid body rotation are the same as for the hydrogen atom with angular momentum L2 = L(L+1)~~h~~2, l = 0,1,2,3,… . Classically, the rotational KE is the total energy E = L2/2I, in this case I is the Rotational Inertia. Therefore the energy eigenvalues are:

 En = L(L+1)~~h~~2/2I .

For a diatomic molecule I = mrr02 , where the **reduced mass**, mr = m1 m2/( m1 + m2).

Vibrational and rotational quantized energy levels exist and transitions in quantum number (and energy) produce molecular spectra. In some situations, vibrational transitions cannot happen (low temperature, for example). In this case, the levels are **frozen out,** and the ground state is E0 = (1/2)~~h~~ω, something we call **zero point energy**.

**41. Atomic Structure.**

**There are four Electron Quantum Numbers**: n, L (= 0, 1,…m=, n-1) , mL = -L to L , and ms = + or – ½. No two electrons can occupy the same state with the same four quantum numbers. This is **The Pauli Exclusion Principle**. This is true of all **fermions** (with spin ½--protons and neutrons too). **Bosons** (with integer spin like photons and gravitons) can share quantum numbers and even be in the same quantum state—we call that a **Bose-Einstein Condensate** .

**Hydrogen** has all its quantum numbers but n **degenerate** (or nearly so), that is, a given n has the same energy levels for all L, mL , ms .

However, **heavier atoms than H** split the sublevels of the primary levels n, **breaking the degeneracy**.

**Electron Configurations:** nLN  . State of L ={s, p, d, f} (speedy fine), N = no. in substate.

**The Zeeman Effect:** The adjacent L (L and L+1) levels are separated by Energy U = μB = (e~~h~~/2m)B, [where μ = IA = (ev/2πr)(πr2) =evr/s, but L = mvr🡪μ = e~~h~~/2m]. By looking at the spectral levels of the sun, for example, we can calculate the magnetic field on the sun’s surface.

**Spin Angular Momentum:** For an electron, Sz = + or –(1/2)~~h~~

But the total spin angular momentum = S = ~~h~~ √[(1/2)(1/2 + 1)] = (3/4)~~h~~ .

**Screening:** For sodium and other elements with single electrons as lone occupants of a shell, En = (13.6 eV) Zeff2/n2 .

**42. Molecules and Condensed Matter.**

**The easy stuff:**

**Democritus**, the Greek, called the smallest indivisible part of an element ‘atomos’, or **atom**. However, the atom is not the smallest part of matter, it is only a useful unit in physics and chemistry. Atoms are composed of **neutrons** and **protons** in the **nucleus** and **electrons** ‘in orbit’ around the nucleus. Neutrons and protons are roughly the same mass, about 1836 times greater than the mass of the electron. Protons have one unit positive electric charge, electrons one unit negative. To make an atom **neutral**, we must have equal numbers of protons and electrons. When we strip electrons from an atom it has a net positive charge and we say it is **ionized**.

The number of protons in an atom or ion is called the **atomic number**, Z. It distinguishes one element from another.

The **atomic mass number**, A, is the number of protons plus the number of neutrons. It represents the approximate mass of the element.

**Think**: Look at the periodic table. Z is the top number above the element symbol. A is the bottom number rounded off.

**# neutrons = A - Z** (approximately–true for an **isotope**–differing in nos. of neutrons).

**Atomic notation**: these two numbers are flipped from what they are in the periodic table. For example Helium is 24He. It has two protons and two neutrons, making an atomic mass of 4.

In Chemistry, atoms of elements are building blocks for making **molecules**, or combinations of atoms. A **compound** is a combination of 2 or more elements in a molecule. The number of atoms of an element in a compound is indicated by the subscript after the **element symbol** in a **chemical formula.**

We can use the atomic masses to determine the ratios of weights in a **chemical reaction**. For example, C + O2 combine in a ratio of 12 to 2x16 = 32, the molecular masses of the reactants, to form CO2.

 **Avogadro’s Principle**: equal volumes of different gases contain the same number of molecules at the same pressure and temperature.

**Antimatter**: identical to matter, except that it has the opposite charge (Not opposite mass sign). An antielectron or positron has a positive charge. A mass of antimatter will annihilate an equal mass of matter, completely changing into energy (Einstein described this equivalence of matter and energy).

**Four states of matter**: (Increasing temperature may break bonds)

 **Solid**–atoms or molecules in a fixed ‘bonded’ position.

 **Liquid**–atoms or molecules free to change position, occasionally bond.

 **Gas**–atoms or molecules free–no bonds.

 **Plasma**–a gas of ions. H plasma would be free protons and electrons.

**Solids**

**Bonds**: **intermolecular** vs. **intramolecular**.

**ionic**–one atom rips electron(s) off another, becomes charged, and attracts the other. Ex. NaCl. (1-5 eV)

**covalent**–atoms share electrons, so as to feel as if they have filled valence shell. Ex. H2. (1-5 ev)

**Hydrogen bond (intermolecular)**–polar molecules attract opposite sign of charge. Ex. water. (<0.5 eV)

**Van Der Waals bond:** (<0.1eV) Intermolecular bonds in dense gases.

Two types of Solids:

 **amorphous**–without form–Ex. sandstone.

**crystalline**–repeated atomic or molecular structure: **unit cell**. Ex. quartz.

**X-ray diffraction**–a way of determining crystal structure by bouncing x-rays

 off successive crystal planes and observing the interference patterns.

 Von Laue and Bragg–experiment and theory.

**Properties of Solids**: density, elasticity, opacity, compressibility, melting point, and etc.

**Average density** = Mass/Volume (**d = M/V**). On an atomic level, it compares the mass of an atom to the volume it takes up. **Osmium** is the **densest** element.

**43. Nuclear Physics.**

 **Nuclear Radius:**  R = R0A1/3 . (closely, where R0 = 1.2 x 10-15 m = 1.2 fm)

Rutherford scattered alpha particles off thin gold foil and proved the positive charges in matter were concentrated in the tiny balls we now call the nucleus of an atom. Electrons with a negative charge were found to be ‘orbiting’ this nucleus.

The nucleus is composed of **baryons** which are either protons (+ charge) or neutrons (no charge).

We label the nucleus with the element symbol, say X and the notation:

 **ZA X** ,

Where A is the **atomic mass = number of baryons**, and Z is the **atomic number = number of protons** (which gives the atom its chemical identity).

In nuclear reaction the top and bottom numbers on either side of the equation are identical. This expresses: 1) **Conservation of nuclear charge**, 2) **conservation of baryon number**.

Atoms with the same Z, but different A (different no. of neutrons) are called **isotopes**. Example 1H (normal hydrogen), 2H (D, deuterium, slowly decays to H) , and 1H (tritium, rapidly decays). We make **heavy water** (D2O) with deuterium. Some of the water in a nuclear power plant gets converted by absorbing free n’s.

There are three major types of nuclear decay:

1. **Alpha Decay**: a nucleus gives off a 24He or 24α , a helium 4 nucleus.

2. **Beta Decay**: a neutron in a nucleus (or free) morphs into a proton, giving off an electron. Energized free electrons are called **beta radiation.** 10e = 10β .

 01n 🡪 11p + -10e + νe(with a bar on top--electron anti-neutrino).

3. **Gamma Decay:** an excited nucleus gives off high energy light, keeping the same number of neutrons and the same number of protons. We indicate the excited nucleus by a \* like U\*.

Nuclear decay happens exponentially: R = R0e-λ t, where R0 is the initial rate of decay, R is the current rate, and λ is the **decay constant,** so a log-log plot will be roughly linear and give λ as the slope. The **half life** of an isotope or particle is T1/2 = ln2/λ. It is the time for half the **parent** to change into the **daughter**. Half life of Carbon 12 is roughly 6,000 years.

**Nuclear fission**: breaking apart of nuclei further down the periodic table than iron.

**Nuclear fusion:** fusing together of lighter nuclei than iron.

Why do some break up and some fuse? **Binding energy/baryon** goes up to iron, then down as you go further down the periodic table.

**Nuclear Binding Energy:** EB = (ZMH + Nmn –AZM)c2 . AZM is the mass of the neutral atom, c2 = 931.5 MeV/u (u = closely the mass of the p and n); there are Z protons (included with MH = mass of hydrogen atom) and N neutrons .

**What is obeyed in a nuclear reaction?** conservation of charge, energy, momentum, angular momentum, and nucleon number.

A **nuclear fission power plant** uses a little less than enough to explode as an atom bomb, a little less than **critical mass**. However, if the control rods are left up, the nuclear material gets very hot and undergoes a **meltdown**. It also leaves behind a dangerous residue, which must be disposed of way underground.

A **nuclear fusion power plant** fuses hydrogen into helium and gives off totally clean energy, with almost no dangerous radioactive residue. When we break even (producing more energy output than input), we could use the energy for electricity, break apart hydrogen and power our cars, producing only water—NO GREENHOUSE GASES! Hybrids still produce over 70% of the greenhouse gases compared to normal cars. Our current cars can easily and inexpensively be converted to hydrogen power. Why buy an electric car? Nuclear fusion puts out roughly 6,000 times more energy than is inputted (IF no energy is wasted).

**44. Particle Physics and Cosmology.**

The **Standard Theory of Elementary Particles** says that there are two families of particles: **quarks and leptons.**

**Quarks** have 1/3 multiples of protonic charge—three of them make up a neutron or a proton. There are six standard quarks. Three quarks with -1/3 +2/3 +2/3 units charge = 1 unit charge for a proton. Three quarks with -1/3 -1/3 +2/3 make up a neutron.

Quarks are tightly bound in **baryons** (neutrons or protons) by the **strong nuclear force**, a contact force which holds protons and neutrons together in the nucleus.

**Leptons** are electron-like particles and their associated neutrinos:

Electron (e) and electron neutrino (νe),

Muon (μ) and muon neutrino (νμ) ,

Tauon and tauon neutrino (ντ).

Each has its own antiparticle.

**Mesons:** the force between nucleons is mediated by mesons, creating potential energy U(r) = -f2e-r/ro/r (nuclear potential energy).

**Photons** mediate the electromagnetic force.

The **superstring theory** claims that quarks and leptons are variations on the

 same thing: a vibrating loop of **string** in multidimensions (now 10).

**M Theory** (membrane theory) says straight strings connect to membranes in 11 dimensions. This unifies 5 possibilities for superstring theory into one.

The above two theories are called **Theories of Everything (TOE).**

Both above theories are considered to be **Quantum Gravity Theories**, integrating Quantum Theory with the General Theory of Relativity.

**Particle characteristics:**

 **Spin:** a particle is either a

 **Boson** with integer spin (x h/2π) (photons and gravitons are examples) or

 **Fermion** with half integer spin (protons, neutrons, and electrons are examples).

 **Mass: m(proton) = 1.67 x 10-27 kg ≈ m(neutron)**

 **m(electron) = (1/1836) m(proton).**

Other characteristics are lepton number, strangeness, etc.

**Particle accelerators** like the **LHC = Large Hadron Collider** in Geneva

 accelerate particles and collide them at high speed so that the kinetic energy can create unusual particles and states (by E = mc2).

 Soon it is hoped that **supersymmetric partners (sparticles)** of particles with large mass may be formed, confirming **Supersymmetry**, which is the foundation of the string theories.

 We may also find the large mass **Higgs Boson**, which in some unifying theories is responsible for the different masses of different particles.

**Cosmology:**

 **The Hot Big Bang Theory** says that in **General Relativity,** space-time could have expanded from an infinitesimally small point. The more compressed it was, the hotter it was, just like compressing a gas heats it.

**Hubble’s Law:** local galaxies move under the influence of gravity, but super-clusters (clusters of clusters) of galaxies not too far away expand with the universe. Recession velocity, v = H0r (r is distance), where v comes from the Doppler shift: v = [(λ0/λS)2 - 1]/ [(λ0/λS)2 + 1] . H0 ≈ 7 (km/s)/Mpc .

 **One Parsec:** 1 pc = 3.26 light years.

Wavelengths expand with the expanding universe: if R is the scale of super-cluster distances, λ/λ0 = R/R0 .

A quantity called **red-shift** is another way of expressing cosmological distance: z = ∆λ/λ = R/R0 - 1 .

However, when we go out about 7 billion light years, we find the linear Hubble’s Law begins to turn down, indicating the universe is accelerating.

**Critical Density:** We can use E = (1/2)mv2 – GmM/R = 0 (the edge of being bound—defining universal escape speed), v = H0/R, and ρc = M/(4/3)πR3 to give us ρc = 3 H02/8πG = 7.9 x 10-27 kg/m3. Amazingly, our observations say that the current density of the universe is ρ = (1.02 +- 0.02)ρc **.**

**Three Observations Supporting the Big Bang:**

 **1. The cosmological red-shift** increases linearly with distance to moderately distant super-clusters of galaxies.

 **2. Nucleo-synthesis**—the Big Bang was hot enough to be like a star for about a minute, fusing 25% of the mass of hydrogen to helium. We currently see this mix in masses gas too sparse to have gathered by gravity to create stars.

 **3**. **The Microwave Background**—electrons combining with protons for the final time at 380,000 years after the Big Bang produce light which stretches to a 2.7 K temperature Black Body radiation curve—microwaves coming to us from every direction. Ultraviolet wavelengths were emitted when electrons recombined with protons to make Hydrogen permanently (before then light was breaking them apart). This is called the **Recombination Era**. These wavelengths were stretched to microwaves by universal expansion.

 **WMAP**, the Wilkinson Microwave Anisotropy Project is a satellite that gives us a picture of the universal density distribution at 380,000 years after the big bang, and gives us most of our important cosmological data. The **Planck Satellite** (to be put up soon) will do an even better job, gives us a much more detailed picture even earlier in universal history.

**The Accelerating Universe:**

Observations of exploding white dwarf stars we call **Supernova 1a’s** give us a better idea of the history of the expansion by giving us precise distances out to 11 billion light years away, implying the universe is currently accelerating, with a density for the **dark energy** we call the **Cosmological Constant, Λ.** It appears to be constant now, with preliminary data, but if the dark energy (funny energy) is not constant, we theoretically call it **Quintessence** (the 5th element).